Bachelor thesis

Modeling and control of redundant manipulators with kinematic couplings at joints

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1. Introduction

The first aim of the thesis is a computer implementation of dynamic models of planar manipulators with single open-loop chains and revolute joints. The second main purpose is an execution of various simulation cases of operational space control, while paying special attention to redundant manipulators with kinematic couplings at joints. What is important, joint coordinates, rigid bodies and frictionless holonomic constraints are assumed.

2. *Articulated-body* algorithm in connection with manipulator dynamics

Characteristics of this recursive algorithm

It is an example of propagation methods in forward dynamics. Mathematical notation of spatial algebra in the planar case is exploited. Thanks to this, the equation of motion for body B_i in the figure shown below has simple form: $\boldsymbol{f}_i = \boldsymbol{I}_i^A \boldsymbol{a}_i + \boldsymbol{p}_i^A$, where $\boldsymbol{f}_i$ is the force transmitted across joint i; $\boldsymbol{I}_i^A$ and $\boldsymbol{p}_i^A$ are defined to be *articulated-body inertia* and bias force for B_i in A_i; $\boldsymbol{a}_i$ is the resulting acceleration of body B_i. The joint i also constraints the transmitted force to satisfy $\boldsymbol{S}_i^{\mathrm{T}} \boldsymbol{f}_i = \tau_i$, where τ_i is the generalized force (torque) of B_i. The proper quantities are propagating to neighbouring bodies until the whole problem is solved. Finally the generalized acceleration $\ddot{q}_i$ for each body B_i is known.

The results from implementation of this algorithm are compared with analogous outcome delivered by a commercial multibody dynamics package. Then quantities $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{G}$ are then calculated from constructed formulas to obtain equation of motion for manipulator: $\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{\tau}$.

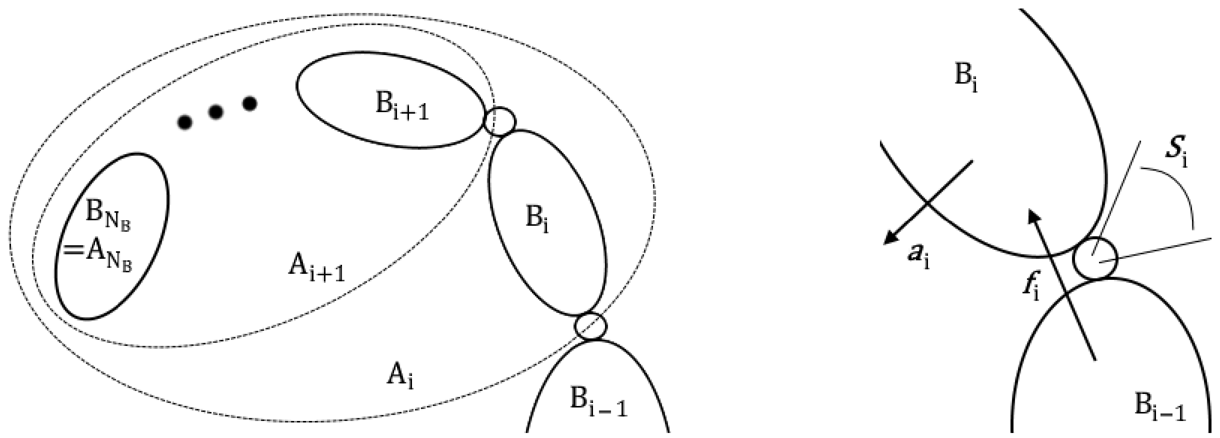

Figure: Primary aspects of the *articulated-body* algorithm: (Left) the way of defining the articulated bodies A_i, (Right) the idea of searching for acceleration $\ddot{q}_i$ corresponding to joint i. N_B is the number of rigid bodies.

3. Operational space control in reference to redundant manipulators

The control objectives are transformed from joint to task coordinates, so that the manipulator's equation of motion has the following form: $\boldsymbol{f}_{\mathrm{T}} = \boldsymbol{M}_x(\boldsymbol{q})\ddot{\boldsymbol{x}} + \boldsymbol{C}_x(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{G}_x(\boldsymbol{q})$, where $\boldsymbol{M}_x$, $\boldsymbol{C}_x$, $\boldsymbol{G}_x$ are received from $\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{G}$ using Jacobian matrix $\boldsymbol{J} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{q}}$ and its derivative $\dot{\boldsymbol{J}}$. In the last formula $\boldsymbol{f}_{\mathrm{T}}$ is the operational space force and $\boldsymbol{x} = [x_1 \;\; x_2]^{\mathrm{T}}$ is the task vector, in which x_1, x_2 are referred to position of manipulator's end effector. The arbitrary generalized force can be expressed as $\boldsymbol{\tau} = \boldsymbol{J}^{\mathrm{T}}\boldsymbol{f}_{\mathrm{T}} + \boldsymbol{N}^{\mathrm{T}}\boldsymbol{\tau}_o$, where $\boldsymbol{\tau}_o$ is its component and $\boldsymbol{N}^{\mathrm{T}}$ is the null space projection matrix. In the case of kinematic couplings at joints, noted as $\boldsymbol{\phi}(\boldsymbol{q}) = \boldsymbol{0}$, mentioned formulas are more complicated. The control law, given by an expression $\ddot{\boldsymbol{x}} = k_p(\boldsymbol{x}_d - \boldsymbol{x}) + k_d(\dot{\boldsymbol{x}}_d - \dot{\boldsymbol{x}})$, is related to proportional-derivative (PD) controller. The quantities $\boldsymbol{x}_d$ and $\dot{\boldsymbol{x}}_d$ (assumed as $\dot{\boldsymbol{x}}_d = \boldsymbol{0}$) refer to goal position and velocity, respectively.

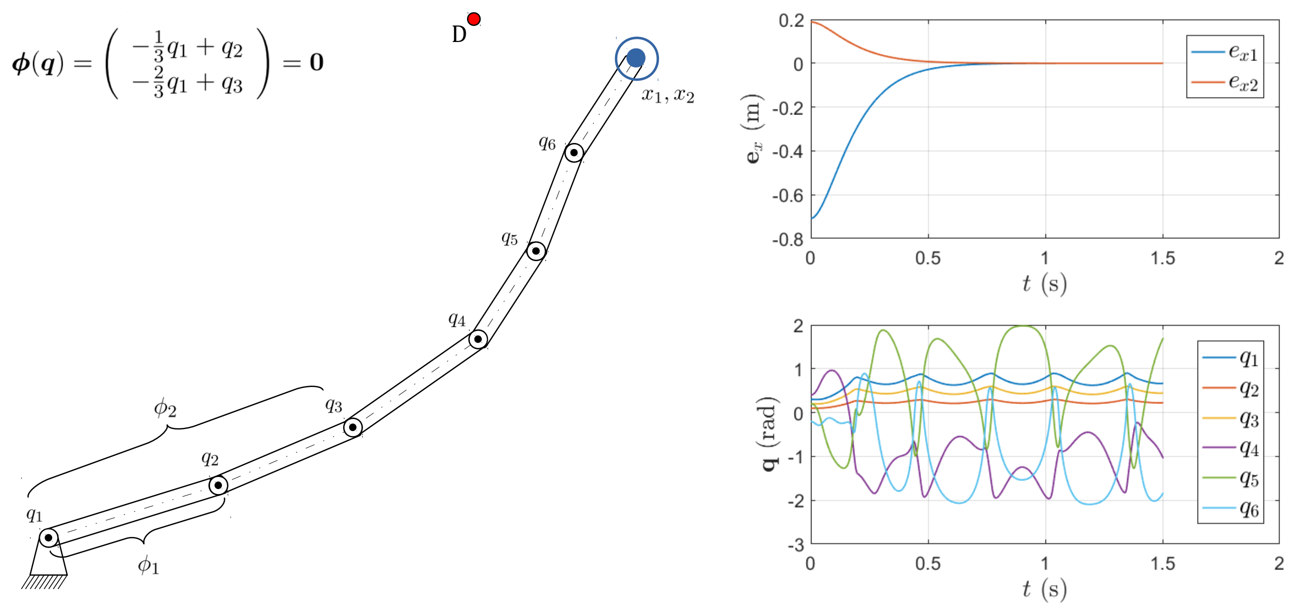

Figure: Operational space control of 6-DOF planar manipulator with two kinematic couplings ϕ_1, ϕ_2 and destination point D. The quantity $\boldsymbol{e}_x = \boldsymbol{x}_d - \boldsymbol{x}$ is the position control error in time t and $\boldsymbol{q}$ is the vector of generalized position variables. $\boldsymbol{\tau}_o = -k_N \nabla U$ is chosen to minimize the gravity effort, defined as $U = \|\boldsymbol{G}(\boldsymbol{q})\|^2$.

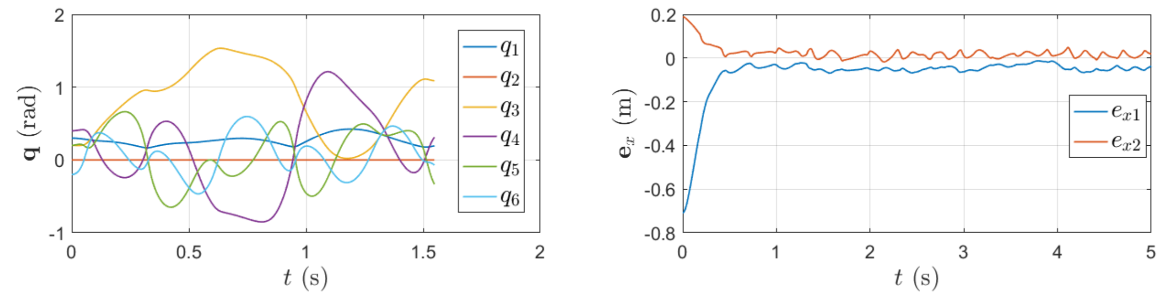

Figure: Two interesting cases of control of 6-DOF planar manipulator: (Left) with the only constraint equation $q_2 = 0$ and $\boldsymbol{\tau}_o = \boldsymbol{0}$, (Right) having assumed $\phi_1, \phi_2, \boldsymbol{\tau}_o = -k_N \nabla U$ and 20% uncertainty in mass of bodies.

4. Conclusions

Articulated-body algorithm has convenient form to be rearranged and used in operational space control, which in case of redundant manipulators enables to achieve goal positions with good accuracy, despite changes of angles q_i. Kinematic couplings at joints reduce variability of q_i or even impose maintaining constant values (e.g. $q_2 = 0$). Uncertainty in physical data, such as mass of bodies, has negative influence on control and makes it more difficult because of nonzero oscillating position error $\boldsymbol{e}_x$.