Master thesis

Optimal control of planar multibody systems using the adjoint method

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Automatics and Robotics
Academic year 2017/2018

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1. Introduction

This paper describes optimal control problem of underactuated planar multibody systems (with more degrees of freedom than controls). The author uses the adjoint and Kelley-Bryson methods, that allow to determine the direction of controls' change that should minimize an objective function.
The optimal control problem is transformed to a optimization problem, that is solved iteratively.

2. Models

The author examines two models mostly used in literature – n inverted pendulums connected in a serial chain and a planar crane with a mass hanged with a rope. System with n pendulums is actuated only with force u acting on a trolley. Crane has two generalized forces: the force F acting on a trolley and the momentum M coiling rope on a crank.

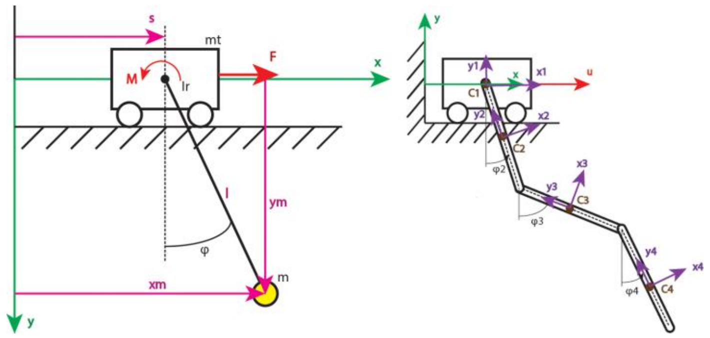

Figure 1: Models of planar crane and triple inverted pendulums.

3. Swinging up inverted pendulums

The work deeply describes swinging up problem of a single, triple and four-fold pendulum. Below the objective function of system with four pendulums is shown. The goal is to find u that will minimize:

$$J = \sum_{i=2}^{5} (\varphi_i(T) - \pi)^2 + \sum_{i=2}^{5} (\omega_i(T))^2 \qquad (1)$$

The objective function (1) include angular position and velocities at the final time $T = 3\ s$.

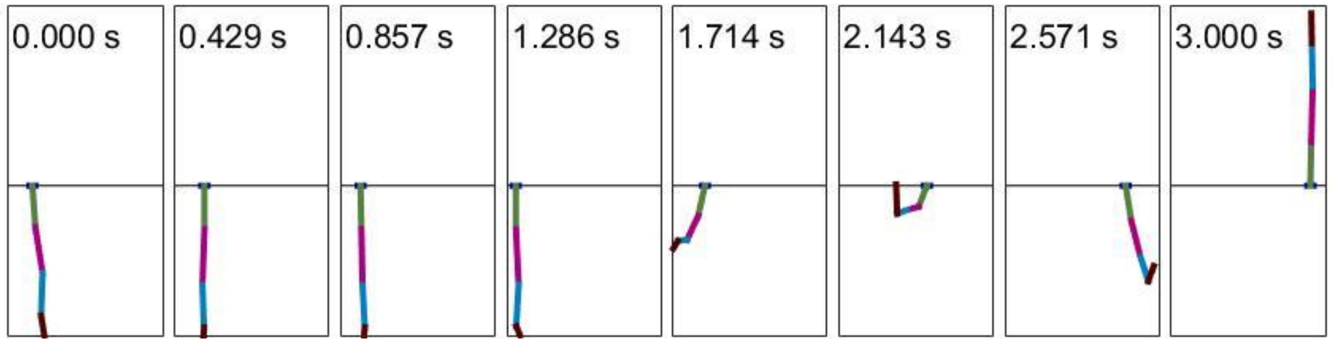

Figure 2: Motion's animation of system with four pendulums in a serial chain.

4. The crane trajectory tracking

$$J = \int_0^3 [(x_m - x_d)^2 + (y_m - y_d)^2]\, dt + (x_m(3) - 5)^2 + (y_m(3) - 1)^2 \qquad (2)$$

The objective function (2) represents trajectory tracking problem. The desired trajectory is described by x_d and y_d. The objective function contains scrap function that is crucial for good convergence of algorithm. The final time is $T = 3\ s$.

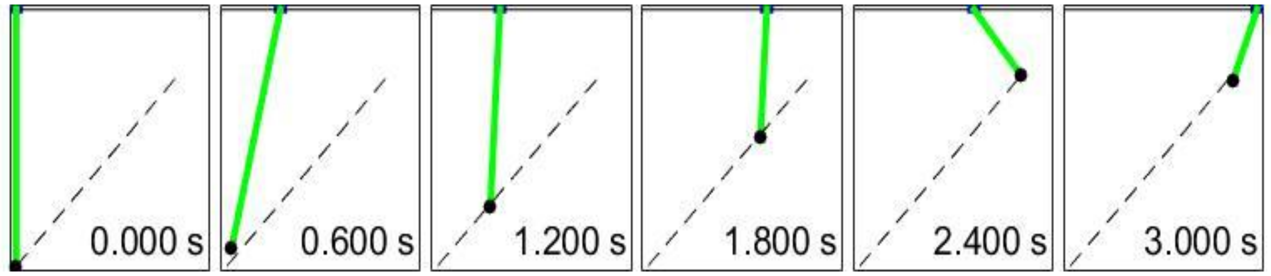

Figure 3: Motion's animation of the crane in trajectory tracking problem.

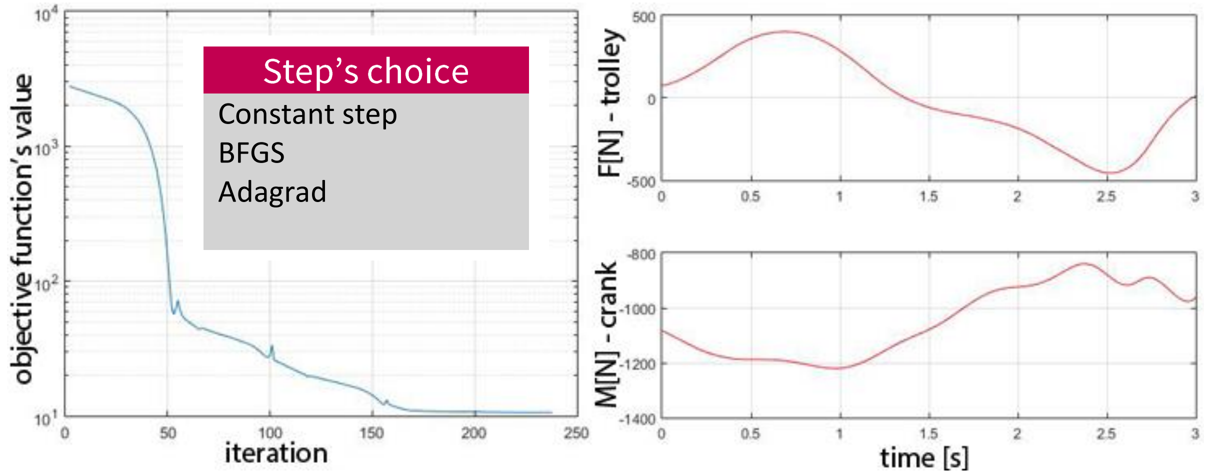

Figure 4: The objective function values (on left) and obtained controls (on right).

5. Conclusions

- The adjoint and Kelley method allow to find optimal control of underactuated system
- The strategy of determining step in each iteration is very important. Large step can omit local minimum, but small one lengthens calculation time
- In this optimization problem local minimums are a serious problem. A step in each iteration must be selected to considered model and used objective function form

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